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Deductibles in health insurance

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ABSTRACT

This study is an extension to a simulation study that has been developed to determine ruin probabilities in health insurance. The study concentrates on inpatient and outpatient benefits for customers of varying age bands. Loss distributions are modelled through the Allianz tool pack for different classes of insureds. Premiums at different levels of deductibles are derived in the simulation and ruin probabilities are computed assuming a linear loading on the premium. The increase in the probability of ruin at high levels of the deductible clearly shows the insufficiency of proportional loading in deductible premiums. The PH-transform pricing rule developed by Wang is analyzed as an alternative pricing rule. A simple case, where an insured is assumed to be an exponential utility decision maker while the insurer's pricing rule is a PH-transform is also treated.

1. Introduction

Health insurance is one of the lines that is largely open to moral hazard and adverse selection. Asheim et al. [1] classify individuals as frail and robust and they claim that frail individuals may try to act as robust in order to get insurance at a lower premium. Insurers have to offer a selection of policies to induce individuals to reveal their state of health. Deductible insurance is considered to be one way to screen out individuals since it is mostly expected that robust individuals will go for deductible insurance. Gardiol et al. [4] in a study they have carried out on the Swiss health insurance system discuss the effect that deductible insurance has on the system and they point out that people who expect a small marginal rate of substitution for health care whether that is due to good health or low risk aversion, will go for a higher deductible.

The choice for deductible insurance depends highly on the degree to which premiums are lowered. The insured acting rationally, will evaluate the expected out-of-pocket payments and there will thus be some loading that he will be indifferent between purchasing or not purchasing the reduction. A well accepted result is that under an actuarially fair premium the optimal choice of the insured is to buy full coverage. Mossin [7] has shown that less than full coverage is optimal where there is a positive loading. Doherty and Schlesinger [3] have extended the subject to where the initial wealth is random, concluding that if initial random wealth is independent of the insurable losses nothing changes from the above.

Mack [6] analyses the relationship between the premiums for deductible cover and full cover in terms of an exponential utility function of the insurer. He proves that if an insurer has an exponential utility function and losses are distributed exponentially then the loading for the deductible cover is an increasing function of the deductible amount. He, however, mentions an exception to the rule. He proves that if the deductible is applied per single loss and losses are independent and identically distributed random variables, then the loading depends on the frequency distribution. If the distribution is

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binomial, Poisson or negative binomial, then the loading is an increasing, constant or decreasing function of the deductible amount.

As an alternative to expected utility, Yaari [12] developed a theory where attitudes toward risk are characterised by a distortion function applied to the probability distribution contrary to expected utility in which the attitude toward risk is measured by a utility function on wealth. Wang [9–11] has developed a pricing rule which is based on a power distortion of the risk distribution. The premium rule known as PH-transform is scale and translation invariant, it satisfies sub-additivity and it produces a value between the expected value and the maximum loss. It also satisfies three desirable properties for layer pricing namely positive loading at any layer, decreasing absolute loading at higher layers and increasing relative loading at higher levels. In a recent paper, Denuit et al. [2] derive most of the classical risk measures using the equivalent utility principle in different theories such as dual utility theory and rank dependent utility.

In this study, we aim at analysing the effect of deductibles on the premium and compare the loss elimination ratio provided by each deductible level across different distributions of the loss variable. We compute the ruin probability under an expected value principle of pricing and evaluate the conditional mean and standard deviation of losses to the insurer.

We analyse the PH-transform as an alternative pricing rule and compare the results, looking at the effect of the assumed distribution on the premiums. We also consider the case where an insured may be an expected utility decision maker, while the insurer may be pricing his product under a Wang transform, and look into whether an equilibrium may be found, without however trying to find any optimal solution.

2. The model

The present study is based on the results of a simulation that was run on a health portfolio of one of the companies in Turkey [8]. Data on 2000 insureds having inpatient and outpatient benefits was analysed, claim frequency and claim amount distributions were fitted to 12 classes of insureds defined depending on the age class, the sex and the type of benefit.

Claim frequency fitted the negative binomial for all the classes, while claim amounts followed the Burr, the loglogistic, the inverse Burr and the inverse Gaussian.

Since the company analysed had no deductible policies, as-if deductibles were introduced to the simulation model. The deductible levels were arbitrarily chosen to start off at 25 NTL (1.7 NTL $\sim$ 1 Euro) and were increased in steps of 25 NTL up to a maximum of 250 NTL for outpatient benefits and correspondingly from 250 NTL to 2500 NTL for inpatient benefits. Risk premiums were derived for each class as the mean loss to the insurer with a linear loading to account for contingencies. The conditional mean and standard deviation of losses as well as the loss elimination ratio at each deductible level were also computed. The simulation was run 1000 times for the whole portfolio and the probability of ruin was derived for different levels of the loading parameter and the initial reserve.

There were several observations on the results which we present below trying to make generalisations whenever possible.

2.1. Observations on the results

Most of our observations derive from the application of the as-if deductibles. When an ordinary deductible is introduced the insurer pays only the part of the losses above the deductible. The risk variable for the insurer is defined by

$$X_D = \begin{matrix} 0 & X \leq D \\ X - D & X > D. \end{matrix} \tag{1}$$

The loss elimination ratio (LER) is defined as the ratio of the decrease in the expected payments of the insurer under a deductible D to the expected value of payments under full coverage and is given by

$$\text{LER} = \frac{E\,[X;D]}{E\,[X]} \quad \text{where} \tag{2}$$

$$E\,[X;D] = \int_0^D xf(x)\mathrm{d}x + D\int_D^\infty f(x)\mathrm{d}x \tag{3}$$

represents the out-of-pocket expenses of the insured.

The above can also be represented in terms of the decumulative distribution function as follows.

Consider X, a non negative loss variable with cumulative distribution function $F_X(x) = \Pr(X \leq x)$. The expected loss can be evaluated from the decumulative distribution function $S_X(x) = \Pr(X > x)$ as

$$E(X) = \int_0^\infty S_X(x)\mathrm{d}x. \tag{4}$$

A proof of the above may be found in [5,10].

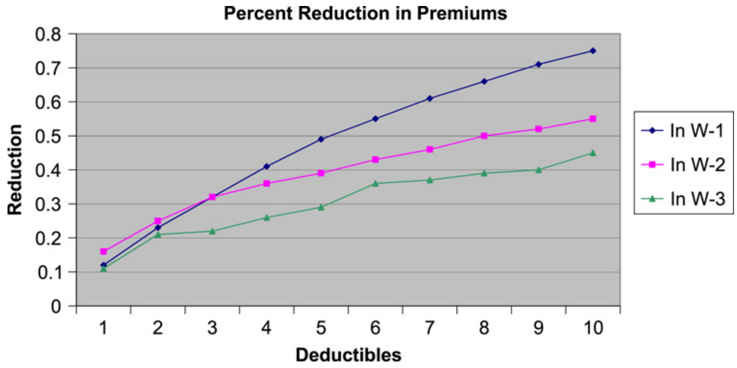

Fig. 1. Loss elimination ratio for inpatient women.

Consider now, the loss variable defined in (1). The expected value given in (3) can be expressed in terms of the decumulative distribution function as

$$E[X; D] = \int_0^D S_X(x)\mathrm{d}x. \tag{5}$$

Proof. Consider $E[X; D] = \int_0^D xf(x)\mathrm{d}x + D\int_D^\infty f(x)\mathrm{d}x$. Replacing the right integral by $DS_X(D)$ and integrating the left integral by parts we get:

$$E[X; D] = \int_0^D xf(x)\mathrm{d}x + DS_X(D) \tag{6}$$

$$= DF_X(D) - \int_0^D F_X(x)\mathrm{d}x + DS_X(D) \tag{7}$$

$$= D(1 - S_X(D)) - \int_0^D [1 - S_X(x)]\ \mathrm{d}x + DS_X(D) \tag{8}$$

$$= \int_0^D S_X(x)\mathrm{d}x. \tag{9}$$

The loss elimination ratio may then be expressed as:

$$LER = \frac{\int_0^D S_X(x)\mathrm{d}x}{\int_0^\infty S_X(x)\mathrm{d}x}. \tag{10}$$

Observation I. *The percentage decrease that each deductible level creates on the premium is a decreasing function of the deductible. This implies that the LER has a declining marginal increase.*

We have computed the loss elimination ratio for all classes of insureds. Fig. 1 gives the results for inpatient women. The above observation has been shown to hold for any arbitrary distribution and hence is independent of whether the analysis is made on an individual or collective basis (see Appendix A for proof).

The decreasing marginal increase of the LER implies that introducing policies with deductibles beyond a certain level might not be too meaningful for the insurer. When looked at from the viewpoint of the insured it implies that the marginal increase of out-of pocket expenditures declines as the deductible increases. This might make the product open to moral hazard since the insured might tend to buy high deductible insurance particularly if he judges the price is low.

Observation II. *The deductible level at which a certain loss elimination ratio is achieved differs across the distribution of losses.*

We compared the LER produced at each deductible level for inpatient and outpatient benefits of men and women of the same age. We present the deductibles as a percentage of the mean loss so as to generate a standard basis of comparison. Table 1 shows the results for four classes of insureds.

The inverse Gaussian and the lognormal were found to best fit data for inpatient women and inpatient men, respectively, while the Burr was found to fit data for both outpatient men and women. It is interesting to note that the standard errors of losses of insureds having the same type of coverage are almost equal. The standard errors are 2.45 and 2.25 for inpatient women and men respectively and 1.25 and 1.013 for outpatient women and men respectively. Highlighting one outcome

Table 1
Loss elimination ratio (LER) across distributions

DED	Inpatient Women II $\mu = 1098; \sigma = 2692$		Inpatient Men II $\mu = 2528; \sigma = 5693$		DED	Outpatient Women II $\mu = 130; \sigma = 163$		Outpatient Men II $\mu = 147; \sigma = 149$	
	Ded/Mean (%)	LER (%)	*Ded/Mean* (%)	LER (%)		*Ded/Mean* (%)	LER (%)	*Ded/Mean* (%)	LER (%)
250	22	16	9	9	25	19	18	17	17
500	46	25	20	15	50	38	34	35	31
750	68	32	30	19	75	58	45	52	39
1000	91	36	39	26	100	77	53	70	47
1250	114	39	49	28	125	96	59	87	54
1500	137	43	59	33	150	115	64	104	58
1750	159	46	69	34	175	135	68	122	60
2000	182	50	79	37	200	154	71	140	65
2250	205	52	89	39	225	173	73	157	64
2500	227	55	99	44	250	192	75	175	69

Table 2
Effect of distribution on deductibles and LER

L^* (%)	DED Expo	DED Pareto	DED	LER (%) Burr	LER (%) Expo	LER (%) Pareto
10	13.70	13.84	25	18	17.4	17.1
20	29.00	26.68	50	34	31.9	30.9
30	46.37	48.10	75	45	43.8	42.0
40	66.40	70.00	100	53	53.6	51.0
50	90.10	96.82	125	59	61.8	58.4
60	119.12	131.03	150	64	68.5	64.5
70	156.52	177.50	175	68	73.9	69.5
80	209.22	247.86	200	71	78.5	73.7
90	299.34	382.50	225	73	82.2	77.2
			250	75	85.3	80.2

we can point out the case where for inpatient men the maximum deductible of 2500 NTL is almost equal to the mean but provides a LER of only 44%, while for women of the corresponding age band, the same deductible corresponds to 227% of the mean loss(thus a very high deductible level) with a loss elimination ratio however of only 55%. The outcomes on the outpatient groups having the same type of loss distribution are almost equal.

To investigate whether the assumed distribution for a particular data set has any effect on the LER, we considered data for outpatient women II, which was found to fit a Burr and assumed an exponential and a Pareto as alternative distributions. We estimated the parameters by the method of moments and we got $\alpha = 0.067$ for the exponential and $(\alpha, \lambda) = (5.88, 634.4)$ for the Pareto. The results are given in Table 2.

The left part of Table 2 gives the deductible for a predetermined loss elimination ratio L^* while the right part is the computation of LER for deductible levels defined for outpatient benefits. Results for the Burr were derived from the simulation. The deductibles for the exponential and the Pareto were derived from

$$D_E = -\frac{1}{\alpha} Ln(1 - L^*) \tag{11}$$

$$D_P = \frac{\lambda(1-K)}{K} \quad \text{where } K = \sqrt[\alpha-1]{(1-L^*)} \tag{12}$$

respectively. The details for the derivation are given in Appendix B.

Observation III. *The conditional standard deviation of losses above the deductible is a non decreasing function of the deductible for all types of insureds.*

The outcome for inpatient women of age group I is given in Fig. 2. We note that the standard deviation of the losses paid by the insurer increases at high deductible levels. The same type of behaviour was seen for all classes of insureds under consideration.

Observation IV. *A constant linear loading for a deductible policy gives rise to an increasing probability of ruin at high deductible levels.*

Ruin probabilities have been computed for 5 different values of the loading parameter, two of which are presented in Fig. 3. It is clear that although the ruin probability is high when no loading is applied, it is not much effected by the deductible level. When a premium with a positive linear loading is applied, the ruin probability drops highly at full coverage but shows

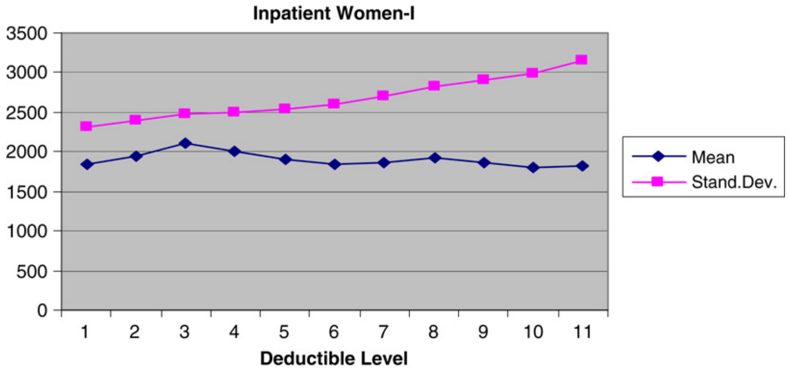

Fig. 2. Mean and standard deviation of losses at each deductible level.

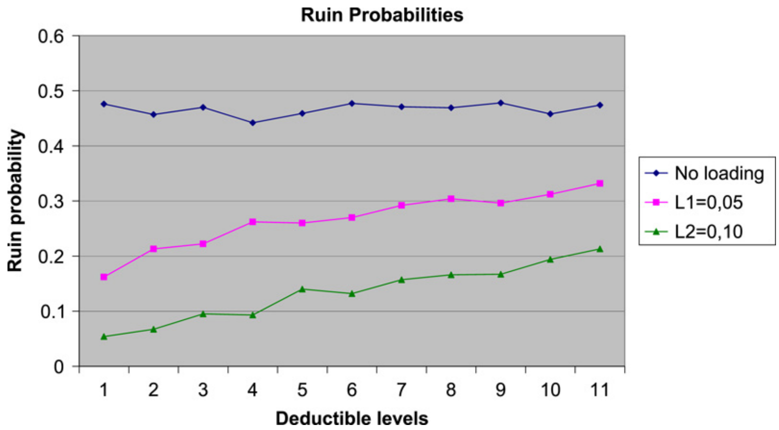

Fig. 3. Ruin probabilities under different values of loading.

an increasing trend as the deductible levels get higher. This may be explained by the increase of the standard deviation of losses at high deductible levels which cannot be compensated for by a constant loading on the premium.

3. An alternative way of pricing

Following Yaari's [12] dual utility theory, Wang in a series of papers [9–11] has presented the proportional hazards (PH) transform which is based on a power distortion of the decumulative distribution function to put more weight on the tail. The distortion produces risk loaded premiums that assume values between the expected value and maximum loss, are translation invariant and satisfy subadditivity.

The PH transform introduced by Wang is a mapping defined by $S_Y(x) = [S_X(x)]^r$ where r is an index defined exogeneously to represent the degree of ambiguity attached on the distribution. The mean of the distribution produces a premium rule given by

$$\pi_r(X) = \int_0^\infty [S_X(x)]^r \, dx \quad 0 < r \le 1. \tag{13}$$

The above is proved in [10]. Following the same steps as in the derivation of statement (5) and using the fact that the mean loss to the insurer is the difference between the expected loss and the out-of-pocket expenditures of the insured we get for the premium rule under a deductible

$$\pi_{rD}(X) = \int_D^\infty [S_X(x)]^r \, dx \quad 0 < r \le 1. \tag{14}$$

Table 3 gives values of the index r for different degrees of ambiguity as cited in [10].

To investigate the effect of the Wang premium principle on our data we considered outpatient women of age group II which had a Burr distribution of losses. Wang [10] shows that if X has a Burr distribution with parameters (α, λ, τ) then the PH transform has also a Burr distribution with parameters $(r\alpha, \lambda, \tau)$.

Table 3
Classification of Wang's index r

Ambiguity level	Index r
Slightly ambiguous	0.960–1.000
Moderately ambiguous	0.900–0.959
Highly ambigous	0.800–0.899
Extremely ambiguous	0.500–0.799

Table 4
Premiums under PH-transform for different values of r

Out-patient deductible (NTL)	$r = 1$ (basic premium)	$r = 0.97$	Equivalent Loading (%)	$r = 0.87$	Equivalent Loading (%)
25	860.17	918.53	6.78	1185.54	37.85
100	497.33	548.79	10.3	706.05	41.97
175	343.96	390.49	13.5	612.70	78.13
250	268.55	308.24	14.8	505.94	88.40

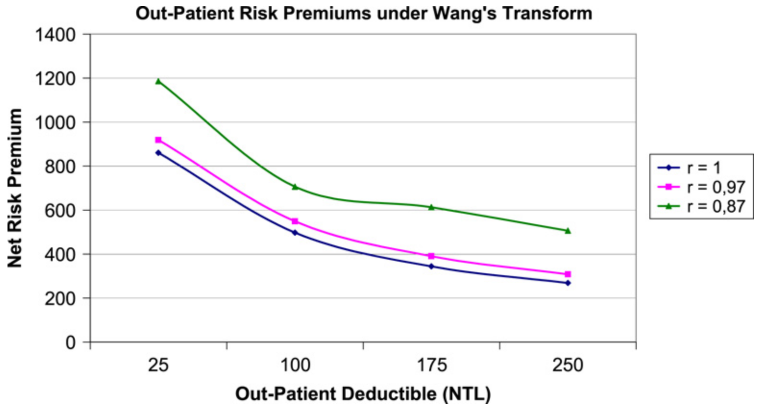

Fig. 4. Effect of Wang transform on premiums.

Table 5
Wang premium rules

Distribution	$S(x)$	Premium rule
Burr	$\left(\frac{\lambda}{\lambda+x^{\tau}}\right)^{r\alpha}$	$\pi_{rB} = \frac{\lambda^{1/\tau}\Gamma(1+\frac{1}{\tau})\Gamma(r\alpha-\frac{1}{\tau})}{\Gamma(r\alpha)}$
Pareto	$\left(\frac{\lambda}{\lambda+x}\right)^{r\alpha}$	$\pi_{rP} = \frac{\lambda}{\alpha r-1}$
Exponential	$e^{-\alpha r x}$	$\pi_{rE} = \frac{1}{\alpha r}$

In our case, the parameters are $(r\alpha, \lambda, \tau) = (0.9224r, 69.8041, 1.9227)$. We used the transformed distribution in our simulation program to get risk adjusted premiums at four deductible levels for two values of the parameter r. The results are given in Table 4 and Fig. 4.

We see from the above that the loading increases with the deductible level in line with one of the properties of the Wang transform mentioned in the introduction. We would therefore expect that there will no longer be an increase in the ruin probability at high deductible levels. We note, however, that the loading implied at high ambiguity levels for high deductibles may produce a price which will be well beyond that which would be acceptable in the market.

We want now to investigate the level at which the premium under the Wang transform is effected across distributions having the same moments. We once more work on the previous three distributions taking as a benchmark the Burr with mean 130 and standard deviation 163. The decumulative distribution function and the premium rule for each distribution are given in Table 5. The premiums computed for a Burr, an exponential and a Pareto having the same moments and the implied loadings are given in Table 6.

The above clearly shows that the same Wang parameter may produce highly different premiums for a set of data assumed to fit different distributions.

Suppose now that we look at the process as a game between two parties namely the insurer and the insured. We may assume that the insurer prices his product following a Wang transform. We might, on the other hand, expect to have insureds evaluating the premium they would like to pay by maximising their utility over wealth since we deem this approach to be more understandable by insureds. The question of whether there can be an equilibrium in the market when the two parties

Table 6
Premiums implied by different values of the Wang parameter r

Wang parameter	Burr premium	Equivalent loading (%)	Pareto premium	Equivalent loading (%)	Exponential premium	Equivalent loading (%)
0.98	134.50	3.5	133.21	2.46	132.55	2.04
0.97	136.96	5.4	134.88	3.75	133.91	3.09
0.95	142.24	9.4	138.33	6.41	136.73	5.26
0.93	148.08	13.9	141.97	9.21	139.66	7.53
0.91	154.57	18.9	145.81	12.16	142.74	9.89
0.87	17.00	30.7	154.15	18.57	149.31	14.94
0.85	179.28	37.9	158.68	22.06	152.82	17.64
0.82	195.84	50.6	166.00	27.70	158.41	21.95

Table 7
Premiums under expected utility and PH-transform for an exponential loss

Type of premium rule	No deductible	With deductible D
Expected utility	$\rho_{cE} = \frac{1}{c} \ln \frac{\alpha}{\alpha-c}$	$\rho_{cDE} = \frac{1}{c} LN\left[1 + \frac{ce^{-\alpha D}}{\alpha-c}\right]$
PH-transform	$\pi_{rE} = \frac{1}{\alpha r}$	$\pi_{rDE} = \frac{1}{\alpha r} e^{-\alpha r D}$

Table 8
Wang parameters corresponding to risk aversion in expected utility

Deductible level	Risk aversion	Wang r
100	0.0005	0.9713 Slight
100	0.0025	0.8434 High
250	0.0005	0.9790 Slight
250	0.0025	0.8799 High

follow a different evaluation procedure then arises. Finding the optimal policy is another question which we do not claim to touch upon here.

4. Expected utility versus Wang transform

We try in this section, to compare premiums defined under expected utility and the PH-transform developed by Wang for a set of risk aversion parameters. We try to find out whether the degree of risk aversion implied matches in the two methodologies.

We carry the analysis in two steps:

(a) We find the premium for a particular risk aversion parameter c under expected utility and solve for the corresponding ambiguity parameter r.
(b) We compute independently premiums under the two approaches for a number of parameters and try to see the relation assuming that both parties accept the same loss distribution as well as the case where the assumption on the loss distribution varies.

The general pricing rule for a risk variable X under expected utility is given in [2] as

$$\rho_c = \frac{1}{c} \ln E(e^{cX}). \tag{15}$$

Table 7 gives the pricing rules under an exponential utility function with an exponential distribution of losses. The same is done for a PH-transform with ambiguity parameter r.

The loss distribution is defined by $f(x) = \alpha e^{-\alpha x}\, x > 0$.

If we denote ρ_{cDE} defined in Table 7, shortly by ρ_c^*, to find the corresponding Wang parameter r we shall have to solve

$$\rho_c^* = \frac{1}{\alpha r} e^{-\alpha r D}. \tag{16}$$

The above equation can be solved on Mat-lab using the Lambert W function (see Appendix C for details).

We have solved the above formula for two levels of the deductible and two levels of the utility risk aversion parameter.

The corresponding Wang parameters are given in Table 8.

A comparison of premiums under the two pricing rules for two levels of the deductible is given in Table 9.

We note that the premiums under both pricing rules given that the risk aversion parameters of both parties are close, fall almost in the same range.

Table 9
Premiums under expected utility and Wang transform

Deductible 100				Deductible 250			
Risk aversion c	EU premium	Wang premium	Wang parameter	Risk aversion c	EU premium	Wang premium	Wang parameter
0.0005	63.40	61.24	1	0.0005	20.21	19	1
0.001	66.94	62.42	0.98	0.001	21.60	20.15	0.98
0.0015	70.92	63.55	0.97	0.0015	23.19	20.75	0.97
0.002	75.42	65.89	0.95	0.002	25.04	22.02	0.95
0.0025	80.56	68.35	0.93	0.0025	27.20	23.37	0.93
		70.94	0.91			24.82	0.91
		76.52	0.87			28.04	0.87
		79.53	0.85			29.83	0.85
		84.37	0.82			32.76	0.82

Suppose now that the insurer assumes the loss distribution is Pareto and applies the same deductibles. The Wang premium for a Pareto under a deductible D may be easily derived as

$$\pi_{rDP} = \frac{\lambda^{r\alpha}}{(r\alpha - 1)(\lambda + D)^{1-r\alpha}}. \tag{17}$$

We can now compute the premiums under the deductible utilising the premium rule defined by (17). We find out that the premiums implied by the above risk aversion parameters are in the range of [63.64–94.88] for a deductible of 100, while for a deductible of 250 the range is [25.69–46.64]. We note that both ranges are well beyond the acceptable premiums of the insureds for the risk aversion parameters considered in Table 9.

5. Discussion

The present study is an exercise looking into the characteristics of some basic notions that we encounter in insurance pricing. Extensions on the present study may evolve in two directions. One may look into real data of companies with deductible policies and compare the results trying to better explain our observations. There were several other observations that came out of the original study which we chose not to present here due to lack of empirical data. One such observation was that out-of-pocket expenses of women were higher than men at all deductible levels which implies that a different price for women may be justifiable. Another observation was that the effect of deductibles at young ages, where the premium is lower, was higher than in older ages. Another point that may need a closer analysis is the conditional standard deviation to see whether it is more affected by the frequency or the severity of claims and whether it depends on the level of the deductible.

The second direction may be to look closer at the case where the insured is an expected utility decision maker while the insurer uses a PH-transform for pricing. The study may be extended to finding optimal pricing rules under the above assumption or make a comparison with other pricing rules. To our knowledge, there is no extensive study comparing pricing rules under deductible insurance.

Appendix A

Proof of Observation I. The results of the simulation implied the marginal increase of the LER was decreasing with high deductible levels. To generalise the empirical observation we proceed as follows:

It was shown in (10) that the loss elimination ratio of the insurer can be expressed as

$$LER = \frac{E\,[X;D]}{E\,[X]} = \frac{\int_0^D S_X(x)\mathrm{d}x}{\int_0^\infty S_X(x)\mathrm{d}x}.$$

We can compute from here the rate of increase of LER with increases in the deductible as

$$\frac{1}{E\,[X]}\frac{\mathrm{d}}{\mathrm{d}D}\int_0^D S_X(x)\mathrm{d}x = \frac{1}{E\,[X]}S_X(D).$$

This implies that the rate of increase of LER is proportional to the survival function evaluated at the deductible D.

To compute the behaviour of the marginal increase of the LER with increasing deductibles we have to differentiate the above expression once more from which we get

$$\frac{1}{E\,[X]}\frac{\mathrm{d}}{\mathrm{d}D}S_X(D) = \frac{1}{E\,[X]}\frac{\mathrm{d}}{\mathrm{d}D}\left[1 - F_X(D)\right] = -\frac{1}{E\,[X]}f(D) < 0$$

and hence the marginal rate of increase of LER is decreasing.

Appendix B

To show the effect of the underlying distribution on the deductible level needed to achieve a certain LER

We consider an exponential and a Pareto distribution with equal moments. The expressions for the deductible in terms of LER for the two distributions are given below.

(a) *The exponential distribution*

Consider an exponential distribution with parameter α. The loss elimination ratio for this distribution is defined by:

$$LER_E = \frac{E\left[X; D\right]}{E\left[X\right]} = \frac{1/\alpha(1-e^{-D\alpha})}{1/\alpha} = (1-e^{-D\alpha}).$$

From here we get, for a specific LER which may be set by the insurer, say L^*, where $0 < L^* < 1$,

$$L^* = (1-e^{-D\alpha})$$
$$Ln(1-L^*) = -D\alpha$$
$$D_E = -1/\alpha Ln(1-L^*).$$

(b) *The Pareto distribution*

Let us now consider a Pareto distribution with decumulative distribution function

$$S_P(x) = \left(\frac{\lambda}{\lambda+x}\right)^{\alpha}.$$

The loss elimination ratio at a deductible level D is given by:

$$\mathrm{LER}_{P=}\frac{\int_0^D \left(\frac{\lambda}{\lambda+x}\right)^{\alpha} \mathrm{d}x}{\frac{\lambda}{\alpha-1}} = \quad 1-\left(\frac{\lambda}{\lambda+D}\right)^{\alpha-1} \quad \text{for } \alpha > 1.$$

Hence solving for the deductible under a prespecified value L^* of LER, we get

$$1-L^* = \left(\frac{\lambda}{\lambda+D}\right)^{\alpha-1}$$
$$\sqrt[\alpha-1]{(1-L^*)} = \frac{\lambda}{\lambda+D}$$

from which we get

$$D = \frac{\lambda(1-K)}{K} \quad \text{where } K = \sqrt[\alpha-1]{(1-L^*)}.$$

Appendix C

Problem

Given a certain risk aversion parameter of an expected utility function compute the corresponding Wang parameter r for an exponential loss function of parameter α.

Solution

The equation we want to solve is

$$\rho_c^* = \frac{1}{\alpha r}e^{-\alpha r D}.$$

From here we get

$$Ln(1/\alpha) - Lnr - \alpha r D = Ln\rho_c^*$$
$$Lnr + \alpha r D = Ln\left(\frac{1}{\alpha\rho_c*}\right) = K.$$

This implies that we have to find a solution for an equation of the form

$$Lnr + c_1 r = K$$

where $c_1 = \alpha D$ and $K = Ln(\frac{1}{\alpha\rho_c*})$ are constants.

The equation may equivalently be represented as

$$re^{c_1 r} = K^* \quad \text{where } K^* = e^K.$$

This equation can be solved on MatLab with Lambert's W function. Mat-Lab gives the solutions as

$$r = \frac{1}{c_1(lambert\,w(c_1 \exp K^*))}.$$

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